Simplified evaluation method for piezoelectric nonlinear vibration under high power operation

高負荷駆動時における非線形圧電振動の簡易的評価法 Yaoyang LIU¹ and Takeshi MORITA¹ (¹Graduate School of Frontier Sciences, The University of Tokyo) 劉 耀陽^{1†},森田 剛¹(¹東京大学院 新領域創成科学)

1. Introduction

In high power ultrasonic devices, the piezoelectric performances is limited by the nonlinear response due to large stress^[1]. At low voltage operation, the piezoelectric properties are described with the conventional equivalent circuit; which fails to explain the nonlinear phenomena.

In our previous study, we found the main source of nonlinearity is the mechanical nonlinearity and an improved model with extra nonlinear terms was proposed to estimate the nonlinearity in Pb(Zr_xTi_{1-x})O₃ (PZT)^[2] and CuO-(K_{0.4}8Na_{0.52})NbO₃ (CuO-KNN) transducers^[3]. By combining the results from the burst mode measurement and impedance measurement under high power operation, the nonlinear coefficients were determined via curve fitting ^[2-3].

In this study, a simpler method for determining the nonlinear coefficients is suggested. Instead of curve fitting, the admittance curves under both high voltage and low voltage were used to calculate the nonlinear coefficients. This method provides a convenient way to estimate the nonlinear level of the piezoelectric materials.

2. Experimental procedure

CuO-KNN $(6.33 \times 2.54 \times 0.99 \text{ mm}^3)$ and PZT transducers (Fuji Ceramics, hard-type: C-203 $43 \times 7 \times 2 \text{ mm}^3$) poled along the thickness direction were utilized, which were excited in longitudinal mode with 31 effect as shown in Fig. 1. CuO-KNN transducers were fabricated via a hydrothermal method ^[4].

Admittance curves were measured via a frequency response analyzer (NF FRA5097). The transducers were driven by the output signal of NF FRA5097 after amplified by a power amplifier (NF HSA4052). In the burst mode measurement^[5] the samples were excited by a burst signal from function generator NF WF1974 amplified by NF HSA4052. The transducer vibration velocity was measured by a Laser Doppler Vibrometer (Polytec NLV-2500). The driving voltage, vibration velocity, and current were monitored and recorded by an oscilloscope (Agilent DSO5034A).

Combining the burst mode and admittance measurement, we found the nonlinear parameter comes from only the mechanical ports. It means the damped capaciter and the force factor can be taken as linear parameters in the case of resonant drive.



Fig. 1 Piezoelectric transducer (31 mode)

In this previous studies^[2-3], the nonlinear coefficients were obtained from the curve fitting procedures. However, this process was complicated and time consuming; therefore, the simplified calculation method is proposed in this study, which can be carried out only with the admittance curves measurements.

3. Results and discussion

The admittance curve of CuO-KNN transducer under 0.5 and 100 V_{p-p} was shown in Fig. 2. In Fig. 2 (a) and (b), comparing with low voltage admittance curve, the high voltage admittance curve has serious deformation; around the resonant frequency the values are not consecutive like other points, which is usually called as jumping phenomenon. In addition, upward and downward sweep measurements gave different results, forming a hysteresis. These behaviors cannot be explained by the conventional equivalent circuit.

In our previous study^[2-3], the nonlinear terms were taken into account and the motional current amplitude i_0 (corresponding to the vibration at the tip of the transducer) and voltage amplitude V_0 satisfy

$$\left(-\omega L i_{0} + \frac{i_{0}}{\omega C_{0}} + \frac{3\xi i_{0}^{3}}{4}\right)^{2} + \left(R_{0} i_{0} + \frac{3}{4}\eta i_{0}^{3}\right)^{2} = V_{0}^{2} \quad (1)$$

$$\theta = \tan^{-1} \left[\left(-\omega L + \frac{1}{\omega C_{0}} + \frac{3\xi i_{0}^{2}}{4}\right) / \left(R_{0} + \frac{3}{4}\eta i_{0}^{2}\right) \right] \quad (2)$$



Fig. 2 Measured (a) admittance amplitude, (b) phase curve of CuO-KNN transducer under 0.5 and 100 V_{p-p} ; and (c) motional current under 100 V_{p-p} calculated from (a) and (b).

Table 1 Obtained parameters from the simplified method and curve fitting.

Sample (method)	A (N/V)	β_0 (s ⁻¹)	$\eta \left(\Omega A^{-2} ight)$	ω_0 (krad/s)	ξ (Ω A ⁻²)	C_d (pF)
CuO-KNN (This method)	0.0189	864	2.08×10 ⁵	2862	-2.39×10 ⁶	96.8
CuO-KNN (Curve Fitting)	0.0189	864	2.08×10 ⁵	2862	-2.39×10 ⁶	96.9
PZT (This method)	0.153	54	5.04×10 ³	227	-9.72×10^{4}	2030
PZT (Curve fitting)	0.152	54	5.07×10 ³	227	-9.57×104	2060

where, ξ and η are two introduced nonlinear coefficients; ω , L, C_0 , and R_0 are angular frequency, equivalent inductance, linear equivalent capacitance, and linear equivalent resistance; and θ is the phase delay between current and voltage. It was found that L and damped capacitance C_d can be taken as constants even under high voltage; which can be determined by resonant point (f_r , Y_r) and anti-resonant point (f_a , Y_a) of low voltage admittance curve shown in Fig. 2 (a) as follows:

$$C_{d} = \sqrt{Y_{a}}\sqrt{4\omega_{a}^{4}Y_{r}^{2} + \omega_{r}^{4}Y_{a}^{2} - 2\omega_{r}^{2}\omega_{a}^{2}Y_{a}^{2} + \omega_{a}^{4}Y_{a}^{2} - \omega_{r}^{2}Y_{a}^{2} + \omega_{a}^{2}Y_{a}^{2} / \sqrt{2}\omega_{a}^{2}}$$

$$R = 1/\sqrt{Y_{r}^{2} - (C_{d}\omega_{r})^{2}}; C = C_{d}\left(f_{a}^{2} - f_{r}^{2}\right)/f_{r}^{2}; L = 1/4\pi^{2}C_{d}\left(f_{a}^{2} - f_{r}^{2}\right)$$
(3)

where, ω_r and ω_a are angular resonant and anti-resonant frequency; respectively. Motional current in Fig. 2 (c) and phase (not shown here) were calculated from admittance curve; as expressed by:

$$i_m = Y_m V = (Y_{total} - j\omega C_d) V_0 \tag{4}$$

The two points (i_1, δ_1) and (i_2, δ_2) indicated in Fig. 2 (c) satisfy Eq. (1)-(2); δ is the phase of the motional current. Put (i_1, δ_1) and (i_2, δ_2) into Eq. (1)-(2) and solve the equations, we have:

$$\xi = \frac{4}{3} \times \frac{\left(V_0 \sin \delta_1 \times \frac{i_2}{\omega_2} + \frac{\omega_1}{\omega_2} L i_1 i_2 - V_0 \sin \delta_2 \times \frac{i_1}{\omega_1} - \frac{\omega_2}{\omega_1} L i_2 i_1\right)}{\left(\frac{i_1^3 i_2}{\omega_2} - \frac{i_2^3 i_1}{\omega_1}\right)}$$
(5)

$$\eta = \frac{4}{3} \times \left(i_2 V_0 \cos \delta_1 - i_1 V_0 \cos \delta_2 \right) / \left(i_1^3 i_2 - i_2^3 i_1 \right)$$
 (6)

The calculated nonlinear coefficients are shown in Table 1 together with the coefficients calculated with the previous curve fitting method.

This calculation results show consistence with the values determined in the fitting process of admittance curves; demonstrating the efficiency of this simplified method.

4. Conclusions

At high power operation, the jumping and hysteresis in the admittance curves indicate the existence of nonlinearity. In this study, the simplified method was proposed for the piezoelectric nonlinear coefficients measurements. This method requires only the feature points with the high voltage admittance curve, and the obtained parameters coinsided with those obtained in the previous researches^[2-3].

References

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