

Moving sound source for FDTD method

FDTD 法における移動音源の表現について

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1. Introduction

Finite difference-time domain (FDTD) method [1] is a most popular numerical method for sound field analysis. In most cases in the FDTD calculation, sound source and receiver are fixed, then the response between them is mainly calculated. However, to analyze the noise of automobiles and trains, or bat sonar mechanism, it is necessary to implement moving sound sources in FDTD method.

In this paper, two methods are proposed as a method of implementing a moving sound source in the FDTD method. One is the direct method in which source waveform is radiated while switching grid points on the moving path to be driven at every time step. The other is a convolution method in which all impulse responses from the sound source position at every time step are calculated, then the source waveform is convoluted while switching the impulse response according to the sound source movement. Formulation and numerical experiments are carried out for a two-dimensional sound field. The numerical accuracy will be compared between the direct method and the convolution method.

2. Theory

2.1 Direct method

In order to implement a moving sound source in the FDTD method, the grid points on the moving path of the sound source are driven switching every time step according to the sound source position. In the two-dimensional case, when a sound source is located between grid points, four adjacent grid points are driven according to the source weighting functions. When the sound source is located at $(x, y) = (x_i + d_x, x_j + d_y)$ as shown in Fig. 1 (a), the position of the sound source on the local coordinate system is expressed as $(\xi_i, \eta_j) = (d_x/\Delta, d_y/\Delta)$ as shown in Fig. 1 (b), where Δ is grid interval. The source weighting functions are given as follows

$$\begin{aligned} w_1 &= [1 + \cos(\pi\xi_i)][1 + \cos(\pi\eta_j)]/4 \\ w_2 &= [1 - \cos(\pi\xi_i)][1 + \cos(\pi\eta_j)]/4 \\ w_3 &= [1 - \cos(\pi\xi_i)][1 - \cos(\pi\eta_j)]/4 \\ w_4 &= [1 + \cos(\pi\xi_i)][1 - \cos(\pi\eta_j)]/4 \end{aligned} \quad (1)$$

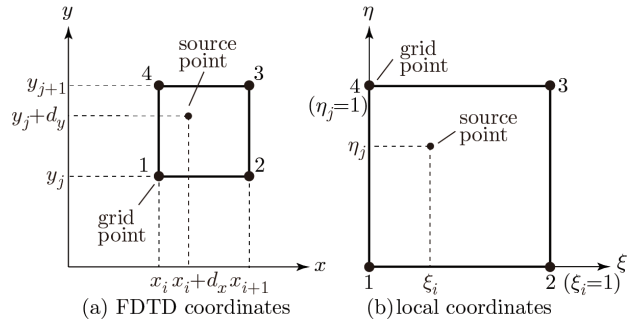


Fig.1 FDTD and local coordinates.

2.2 Convolution method

In order to implement a moving sound source in the FDTD method, we here consider a discrete-time system. When sound source and receiving point are fixed, the acoustic signal p received at the receiving point is expressed as

$$p(k) = \sum_{m=0}^k s(m)h(k-m) \quad (2)$$

where $s(m)$ is sound source signal $h(k)$ is impulse response. In the case of the moving sound source, the positional relationship between the sound source and the receiving point changes every time step, so that the impulse response $h(k)$ also changes accordingly. When the sound source is located at the position $\mathbf{r}(k)$ at a certain discrete time k , the impulse response radiated from that sound source position is expressed as $h(k, \mathbf{r}(k))$. Therefor the receiving signal is expressed as

$$p(k) = \sum_{m=0}^k s(m)h(k-m, \mathbf{r}(m)) \quad (3)$$

Thus, in order to obtain the signal $p(k)$ at the receiving point, it is necessary to obtain $h(k, \mathbf{r}(m))$ at all sound source positions $\mathbf{r}(m)$ on the moving path in advance as shown in Fig. 2 (a). In the case of the FDTD method, multiple impulse responses can be easily obtained by radiating impulse from a receiving point position and calculating responses at

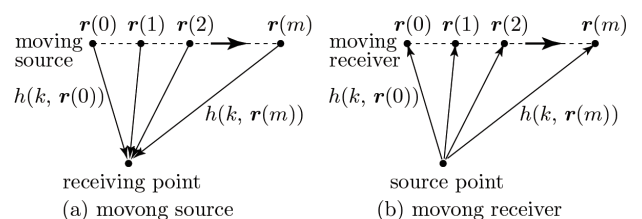


Fig.2 Reciprocity of impulse responses between moving sound source and receiving point.

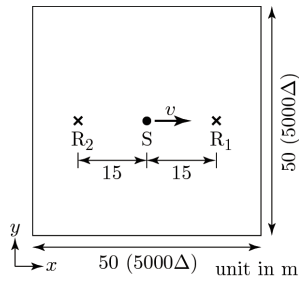


Fig. 3 Numerical model for moving sound source.

multiple sound source positions by the use of reciprocity as shown in Fig. 2 (b).

3. Numerical experiments

3.1 Direct method

Numerical experiments are performed by the CE-FDTD (IWB) method [2,3]. Figure 3 shows the numerical model. The grid size is $\Delta=10$ mm, time step is $\Delta t=29.4 \mu s$, and sound speed is $c_0 = 340$ m/s, so the Courant number χ is 1. The region is divided into $5,000 \times 5,000$ FDTD cells. The boundary condition is Higdon's second order absorbing boundary. The sound source is a point source moving horizontally with constant speed v . Figure 4 shows the calculated sound pressure waveforms at R_1 and R_2 . In these figures, the source speed is represented as Mach number ($M=0, 0.4, \text{ and } 0.8$). The waveforms are compressed by the Doppler effect at the receiving point R_1 in the moving direction of the sound source, and the amplitude increases accordingly. On the other hand, in the reverse direction, they are stretched and the amplitude decreases at R_2 . The numerical dispersion error appears as the sound source speed increases at R_2 . Figure 5 shows the Doppler shift against Mach number M . A 20 cycles burst wave with a frequency of 500 Hz is radiated from the moving source, and the Doppler shift is calculated by FFT of the calculated sound pressure waveform at R_1 and R_2 . The calculation results agree well with

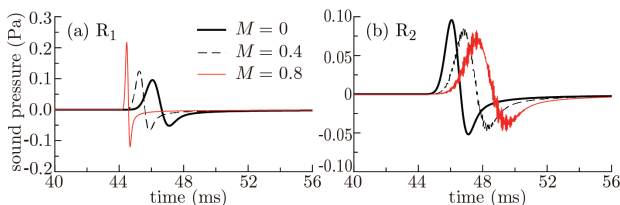


Fig. 4 Sound pressure waveforms (direct method).

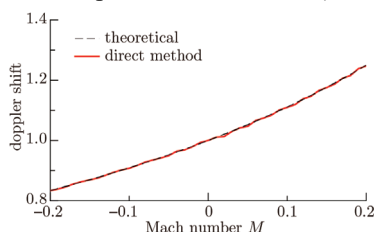


Fig. 5 Doppler shift (direct method).

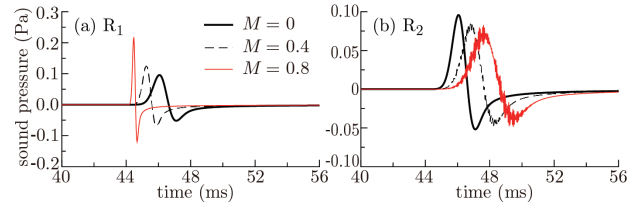


Fig. 6 Sound pressure waveforms (convolution method).

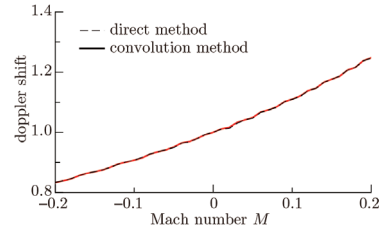


Fig. 7 Doppler shift (convolution method).

the theoretical values, and it is found that the direct method can represent moving sound source with Doppler effect.

3.1 Convolution method

The same numerical experiments with the direct method are performed using the convolution method. Since the sound source starts moving from the center of the region and moves a distance of 10 m, a total of 2,002 impulse responses are calculated in advance between the 1,001 grid points on the moving path and the sound receiving points R_1 and R_2 . Figure 6 and 7 show the sound pressure waveforms and the Doppler shift against Mach number M by the convolution method. Compared with the results of the direct method (Fig. 4, 5), the difference is not apparent on the figure. Figure 8 shows the calculation time against the sound source speed. The convolution method can be calculated in the same time without depending on the source speed, but the direct method shows that the calculation time increases in inverse proportion to the moving speed. It is found that the convolution method can be calculated faster at most source speeds.

References

1. K. S. Yee, IEEE Trans. on Antennas and Prop., **14**, 4, pp.302-307(1966).
2. K. Kowalczyk et al.: IEEE Trans. Audio Speech and Lang. Process., **18** (2010) 78.
3. T. Ishii, T. Tsuchiya, and K. Okubo: Jpn. J. Appl. Phys., **52** (2013) 07HC11.

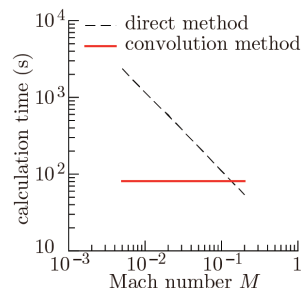


Fig. 8 Calculation time against source speed.