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High-order FDTD Method for Room Acoustic Simulation

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1. Introduction

Sound field rendering exhibit numerical methods to model sound propagation phenomena in spatial and time domain, and are required widely in many engineering and scientific applications. Nowadays, many algorithms have already developed for sound field rendering in room acoustics, especially FDTD schemes due to their high accuracy and ease parallelism. However, the sound field rendering system with traditional second-order FDTD schemes is computation-intensive and memory-intensive as the problem size is increased because oversampling in spatial grids is required to suppress numerical dispersion. Generally, the computing power of solving such wave equation increases as the fourth power of frequency [1][2], and is proportional with the volume of sound spaces. Given the auditory range of humans (20Hz-20kHz), analyzing sound wave propagation in a space corresponding to a concert hall or a cathedral (e.g. volume of 10000-15000 m^3) for the maximum simulation frequency of 20 kHz requires petaflops of computing power and terabytes of memory. This requires computer systems have huge computation capacity and large memory bandwidth.

Many research works were done to reduce the inherent dispersion in FDTD schemes and oversampling in spatial grids, such as digital waveguide mesh topologies [3][4], Interpolated wideband scheme (IWB) [5], high-order explicit “large-star” stencils [6], fourth-order accurate explicit and implicit FDTD schemes [7], and two-step explicit FDTD schemes with high-order accuracy [8]. In this research, the large-star stencils and IWB scheme will be analyzed and their implementations will be discussed.

2. High-order FDTD Scheme

Sound wave propagation in a cubic space is governed by the equation.

$$\frac{\partial^2 P}{\partial t^2} = c^2\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2}\right) \qquad (1)$$

where P denotes sound pressure, c is the speed in air, t is time, x, y, and z are Cartesian coordinates in 3D. To solve equation (1), approximation is usually applied on the partial derivative by using Taylor expansion or polynomial fitting. Thus, a higher-order approximation may achieve high accuracy, reduce the dispersion error, and increase the valid bandwidth. The IWB adopted second-order approximation [5] while the high-order explicit large-star stencils employed Lagrange polynomial fitting [6]. In the large-star schemes, only the grids along the grid axes are taken into the updated equation. For example, in the fourth-order scheme, the partial derivative is approximated by using the following equation [6].

$$\begin{aligned}
\frac{\partial^2 P}{\partial t^2} &= \frac{P^{n-1}_{i,j,k} - 2P^n_{i,j,k} + P^{n+1}_{i,j,k}}{\Delta t^2} \\
\frac{\partial^2 P}{\partial x^2} &= \frac{-\frac{1}{12}(P^n_{i-2,j,k} + P^n_{i+2,j,k}) + \frac{4}{3}(P^n_{i-1,j,k} + P^n_{i+1,j,k}) - \frac{5}{2}P^n_{i,j,k}}{\Delta x^2} \\
\frac{\partial^2 P}{\partial y^2} &= \frac{-\frac{1}{12}(P^n_{i,j-2,k} + P^n_{i,j+2,k}) + \frac{4}{3}(P^n_{i,j-1,k} + P^n_{i,j+1,k}) - \frac{5}{2}P^n_{i,j,k}}{\Delta y^2} \\
\frac{\partial^2 P}{\partial z^2} &= \frac{-\frac{1}{12}(P^n_{i,j,k-2} + P^n_{i,j,k+2}) + \frac{4}{3}(P^n_{i,j,k-1} + P^n_{i,j,k+1}) - \frac{5}{2}P^n_{i,j,k}}{\Delta z^2}
\end{aligned} \qquad (2)$$

Letting $\Delta x = \Delta y = \Delta z = \Delta l$ and inserting equation (2) into equation (1), we have the updated equation for the fourth-order scheme.

$$\begin{aligned}
P^{n+1}_{i,j,k} = \chi^2[&-\frac{1}{12}(P^n_{i-2,j,k} + P^n_{i+2,j,k} + P^n_{i,j-2,k} + P^n_{i,j+2,k} \\
&+ P^n_{i,j,k-2} + P^n_{i,j,k+2}) + \frac{4}{3}(P^n_{i-1,j,k} + P^n_{i+1,j,k} + P^n_{i,j-1,k} \\
&+ P^n_{i,j+1,k} + P^n_{i,j,k-1} + P^n_{i,j,k+1})] + (2 - \frac{15}{2}\chi^2)P^n_{i,j,k} - P^{n-1}_{i,j,k}
\end{aligned} \qquad (3)$$

where $\chi = c\Delta t/\Delta l$ denotes the Courant number. The similar derivation may be applied on the six-order scheme and the updated equation is shown in equation (4).

$$\begin{aligned}
P^{n+1}_{i,j,k} = \chi^2[&\frac{1}{90}(P^n_{i-3,j,k} + P^n_{i+3,j,k} + P^n_{i,j-3,k} + P^n_{i,j+3,k} \\
&+ P^n_{i,j,k-3} + P^n_{i,j,k+3}) - \frac{3}{20}(P^n_{i-2,j,k} + P^n_{i+2,j,k} + P^n_{i,j-2,k} \\
&+ P^n_{i,j+2,k} + P^n_{i,j,k-2} + P^n_{i,j,k+2}) + \frac{3}{2}(P^n_{i-1,j,k} + P^n_{i+1,j,k} + P^n_{i,j-1,k} \\
&+ P^n_{i,j+1,k} + P^n_{i,j,k-1} + P^n_{i,j,k+1})] + (2 - \frac{49}{6}\chi^2)P^n_{i,j,k} - P^{n-1}_{i,j,k}
\end{aligned} \qquad (4)$$

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Equations (3) and (4) show that to update sound pressure of a grid needs the sound pressures of its neighbor grids along axes at previous two time steps .

3. Experiment Results

To verify and estimate the performance of the proposed high-order FDTD schemes, sound propagation in a three-dimensional shoebox with 8m × 8m × 7m was analyzed. To simplify computation, sound pressures of grids on the six boundaries are clamped to 0. The sound field rendering system is designed using C++ programming language, and executed on a desktop with 256 GB DDR4 memories and an Intel Xeon Gold 6212U processor running at 2.4 GHz. The processor contains 24 cores. As comparison, the rendering system based on the IWB scheme are also developed. All the reference C++ codes are compiled by the gcc with option -O3 and -fopenmp to use all the 24 processor cores. During analysis, the sound speed is 340 m/s, the sampling rate of sound is 44.1 kHz, the computed time steps are 1000, and the incidence is an impulse.

3.1 Memory consumption

During computation, sound pressures of grids at previous one and two time steps (time steps n and $n-1$) are stored in memory. Therefore, the required memory directly corresponds to the number of grids, which is determined by the spatial grid size. From the equation of Courant number $\chi = c\Delta t/\Delta l$, Δl equals $c\Delta t/\chi$. When c and Δt are fixed, Δl will be decreased and the number of nodes is increased as the Courant number χ is increased. Table 1 shows the number of grids and the memory demand at each scheme when data are single-precision floating-point and χ is the maximum based on the system stability.

Table 1. Memory consumption

Scheme	Dimensions	Number of grids	Memory (GB)
2nd	599×599×524	188011724	1.504093792
4th	518×518×453	121550772	0.972406176
6th	487×487×426	101033994	0.808271952
IWB	1037×1037×907	975359683	7.802877464

In Table 1, as the order is increased, the grid dimensions are reduced because the Courant number χ becomes smaller and smaller. Thus, the spatial grid size becomes larger. In addition, the grid size in the large-star scheme is much larger than that of the IWB. For example, the grid size in the 6th-order scheme is almost 2.1 times as that in the IWB. This results in the required memory in the IWB is about 9.6 times as that in the 6th-order scheme.

3.2 Computation time and updated throughput

Table 2 presents the average computation time at each time step and the updated throughput. The updated throughput is the number of grids updated per second in each time step. In Table 2, the computation time in the IWB scheme is the longest due to its largest amounts of grids and operations required to update a grid. In the high-order large-star scheme, the computation time per time step in the fourth-order is the shortest. Although the computation in the second-order schemes (2nd-order large-star and IWB) is slow, the updated throughput is not worse because only three x-y planes in the z direction are required and the memory accesses are reduced.

Table 2. Computation time and updated throughput

Scheme	Computation time per time step (s)	updated throughput (G grids / s)
2nd	0.150113	1.252467
4th	0.127764	0.951373
6th	0.129696	0.779009
IWB	1.063461	0.917156

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